

1 Competing phases, strong electron-phonon interaction, and superconductivity in elemental 2 calcium under high pressure

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7 The observed “simple cubic” (sc) phase of elemental Ca at room temperature in the 32–109 GPa range is,
8 from linear-response calculations, dynamically unstable. By comparing first-principles calculations of the
9 enthalpy for five sc-related (nonclose-packed) structures, we find that all five structures compete energetically
10 at room temperature in the 40–90 GPa range, and three do so in the 100–130 GPa range. Some competing
11 structures below 90 GPa are dynamically stable, i.e., no imaginary frequency, suggesting that these sc-derived
12 short-range-order local structures exist locally and can account for the observed (average) “sc” diffraction
13 pattern. In the dynamically stable phases below 90 GPa, some low-frequency phonon modes are present,
14 contributing to strong electron-phonon coupling as well as arising from the strong coupling. Linear-response
15 calculations for two of the structures over 120 GPa lead to critical temperatures in the 20–25 K range as is
16 observed, and do so without unusually soft modes.

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19 I. INTRODUCTION

II. COMPARISON TO RELATED METALS

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20 One of the most unanticipated developments in supercon-
21 ducting critical temperatures (T_c) in the past few years has
22 been achievement of much higher values of T_c in elemental
23 superconductors by the application of high pressure, and that
24 these impressive superconducting states evolve from simple
25 metals (not transition metals) that are nonsuperconducting at
26 ambient pressure. The first breakthrough arose in Li, with T_c
27 approaching^{1,2} 20 K, followed by yttrium^{3,4} at megabar pres-
28 sure also superconducting up to 20 K and showing no sign of
29 leveling off. Both of these metals have electron-phonon (EP)
30 coupled pairing, according to several linear-response
31 calculations^{5–8} of the phonon spectrum, EP coupling (EPC)
32 strength, and application of Eliashberg theory. These impres-
33 sive superconductors have been surpassed by Ca, with T_c as
34 high as 25 K reported⁹ near 160 GPa. Perhaps more unusual
35 is the report, from room-temperature x-ray diffraction
36 (XRD), of a simple cubic (hence far from close-packed)
37 structure over a volume reduction of 45 → 30% (32–109
38 GPa). Whether these two unique phenomena are connected,
39 and in what way, raises fundamental new issues in an area
40 long thought to be well understood.

41 Face-centered cubic (fcc, Ca-I) at ambient pressure, cal-
42 cium transforms at room temperature to body-centered cubic
43 (bcc, Ca-II) at¹⁰ 20 GPa, is identified as simple cubic (sc,
44 Ca-III) in the very wide 32–109 GPa range as mentioned
45 above and shows additional phases (Ca-IV, Ca-V) at even
46 higher pressures. A sc structure for an element is rare, occur-
47 ring at ambient pressure only in polonium and under pressure
48 only in a handful of elemental metals.^{11,12} This identification
49 of a sc structure for Ca is particularly problematic, since it
50 has been shown by linear-response calculations of the pho-
51 non spectrum by a few groups^{13–15} that (at least at zero tem-
52 perature) sc Ca is highly unstable dynamically at all volumes
53 (pressures) in the region of interest. Since these calculations
54 are reliable for such metals, there are basic questions about
55 the “sc” structure itself.

57 Strontium, which is isovalent with Ca, like Ca supercon-
58 ducts under pressure and undergoes a series of structural
59 transitions from close-packed structure to nonclose-packed
60 structure at high pressure. Sr transforms from a fcc phase to
61 a bcc phase at 3.5 GPa and then transforms to Sr-III at 24
62 GPa, to Sr-IV at 35 GPa, and to Sr-V at 46 GPa.¹⁶ The Sr-III
63 structure was first believed to be a distorted sc and later
64 found to be an orthorhombic structure.¹⁷ However, later ex-
65 periments have found that there are two phases coexisting in
66 the Sr-III phase, namely, a tetragonal phase with a distorted
67 β -tin structure and an unidentified additional phase.¹⁷ The
68 Sr-IV structure is very complex and was shown recently to
69 be a monoclinic structure with the Ia space group and 12
70 atoms per unit cell.¹⁸ The structure is more complex in Sr-V
71 and was identified as an incommensurate structure similar to
72 that of Ba-IV.¹⁹ Sr begins to superconduct at 20 GPa, its T_c is
73 8 K at 58 GPa and is believed to be higher beyond 58 GPa.¹⁶

74 Scandium, with one more ($3d$) electron than Ca, under-
75 goes phase transitions from hcp to Sc-II at 20 GPa and to a
76 Sc-III phase at 107 GPa.^{4,20} Although Sc is conventionally
77 grouped together with Y and the lanthanide metals as the
78 rare-earth metals, due to their similarities in their outer elec-
79 tron configurations, its structural transition sequence is rather
80 different from the common sequence of lanthanide metals
81 and Y, which follow the pattern hcp → Sm-type → dhcp
82 → fcc → distorted fcc. The Sc-II structure is complex and
83 was recently found to be best fitted to a pseudo-bcc structure
84 with 24 atoms in the unit cell.²⁰ The structure of Sc-III is not
85 identified to date. Sc begins to superconduct at 20 GPa. Its T_c
86 increases monotonically to 19.6 K with pressure to 107 GPa.
87 Its T_c drops dramatically to 8 K at the phase transition from
88 Sc-II to Sc-III around 107 GPa.⁴

89 Considering the close relation of Sc and Sr to Ca in the
90 periodic table and the similar superconducting properties un-
91 der pressure, it could be expected that Ca under pressure
92 should have more complex structures, rather than the ob-

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93 served sc structure. In fact, Olijnyk and Holzapfel¹⁰ observed
 94 that their Ca sample transformed from sc to an unidentified
 95 complex structure at 42 GPa.

96 So far the higher pressure phases Ca-IV and Ca-V have
 97 attracted the most attention, and considerable progress has
 98 been made in identifying these phases through a combination
 99 of experimental^{9,21,22} and theoretical^{23–25} work. However,
 100 satisfactory agreement between experimental and theoretical
 101 work is still lacking. Ca-IV is identified as a *Pnma* space
 102 group by Yao *et al.*²³ but $P4_32_12$ symmetry by Ishikawa *et*
 103 *al.*²⁴ and Fujihisa *et al.*²² Ca-V seems clearly to have a *Cmca*
 104 space group,^{22–24} however, the calculated enthalpy in the
 105 *Pnma* structure is much lower than in other structures (in-
 106 cluding *Cmca* structure) at pressures over 140 GPa. Also in
 107 the experimental work of Fujihisa *et al.*,²² the fitting of their
 108 XRD patterns to the anticipated $P4_32_12$ and *Cmca* space
 109 groups were not satisfactory and other possibilities still exist.
 110 In the recent work of Arapan *et al.*,²⁵ an incommensurate
 111 structure similar to Sr-V and Ba-IV structures was proposed
 112 for Ca-V phase. Therefore the nature of the Ca-IV and Ca-V
 113 phases is still not fully settled.

114 While helping to forge an understanding the structure of
 115 Ca-IV and Ca-V and its impressive superconducting T_c is
 116 one goal of the present work, our focus has been to under-
 117 stand the enigmatic sc Ca-III phase where relatively high T_c
 118 emerges and increases with pressure, a phase that XRD at
 119 room temperature (T_R) identifies as primitive simple cubic.²¹
 120 In this pressure range sc Ca becomes favored over the more
 121 closely packed fcc and bcc structures, but the dynamical
 122 (in)stability was not calculated by Ahuja *et al.*²⁶ We report
 123 here first-principles calculations of the enthalpy of five crys-
 124 tal structures (with space groups sc, $I\bar{4}3m$, $P4_32_12$, *Cmca*,
 125 and *Pnma*), and linear-response calculations of EPC, that
 126 helps to clarify both the structural and superconducting ques-
 127 tions.

128 III. THEORETICAL APPROACH

129 A. Competing structures

130 The most unstable modes of sc Ca are transverse [001]-
 131 polarized zone boundary modes along the (110) directions. A
 132 linear combination of the eigenvectors of this mode at differ-
 133 ent zone boundary points leads to a body-centered four-atom
 134 cell in the space group $I\bar{4}3m$, whose local coordination is
 135 shown in the cubic cell in the inset of Fig. 1, and has a clear
 136 interpretation as a buckled sc lattice. This structure, when
 137 relaxed, has no dynamical instabilities.

138 The $I\bar{4}3m$ structure is just one kind of distortion from the
 139 sc structure. There are many kinds of other possible distor-
 140 tions. Actually several other structures including *Pnma*,
 141 *Cmca*, and $P4_32_12$ were proposed for the high-pressure
 142 Ca-IV and Ca-V phases.^{22–25} Their structural details are
 143 listed in Table I and their structures are pictured in Refs.
 144 22–24. $I\bar{4}3m$ is a body-centered cubic structure, *Pnma* and
 145 *Cmca* Ca are orthorhombic, and $P4_32_12$ has a tetragonal
 146 symmetry. All are closely related to sc structure. For ex-
 147 ample, $I\bar{4}3m$ turns to simple cubic if $x=0.25$ and the *Cmca*
 148 structure becomes a sc structure if $a=b=c$ and $y=z=0.25$.

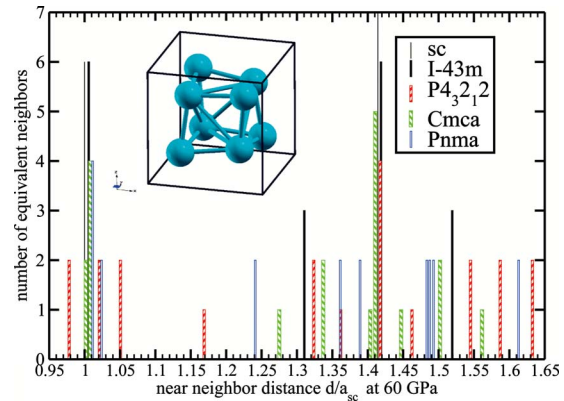


FIG. 1. (Color online) Local coordination of the five structures of Ca, plotted as number of neighbors versus the distance d relative to the cubic lattice constant a_{sc} with the same density. The inset shows the unit cube of the $I\bar{4}3m$ structure (which contains two primitive cells); this structure retains six near neighbors at equal distances but three different second neighbor distances. The $P4_32_12$ and *Pnma* structures can be regarded to be seven coordinated, albeit with one distance that is substantially larger than the other six.

B. Computational methods

We have used the full-potential local-orbital (FPLO)
 code,²⁷ the full-potential linearized augmented plane-wave
 (FPLAPW)+local orbitals method as implemented in
 WIEN2K,²⁸ the QBOX code²⁹ and the PWSCF code³⁰ to do vari-
 ous structural optimizations and electronic-structure calcula-
 tions, and check for consistency among the results. For the
 enthalpy calculations we used the PWSCF code.³⁰ Both QBOX
 and PWSCF use norm-conserving pseudopotentials, while the
 FPLO and WIEN2K codes are all-electron and full-potential
 codes. The linear-response calculations of phonon spectra
 and electron-phonon spectral function $\alpha^2F(\omega)$ were done us-
 ing the all-electron, full-potential LMTART code.^{31,32}

The parameters used in PWSCF for the structural optimiza-
 tions and enthalpy calculations were: wave-function plane-
 wave cut-off energy of 60 Ry, density plane-wave cut-off
 energy of 360 Ry, k mesh samplings (respectively, number of
 irreducible k points) $24 \times 24 \times 24$ (455), $32 \times 32 \times 32$ (897),
 $24 \times 24 \times 8$ (455), $24 \times 24 \times 24$ (3614), and $24 \times 32 \times 32$
 (6562) for sc, $I\bar{4}3m$, $P4_32_12$, *Cmca*, and *Pnma* structures,
 respectively. Increasing the number of k points lowers the
 enthalpy by only 1–2 meV/Ca almost uniformly for all struc-
 tures, resulting in negligible change in volume, lattice con-
 stants, and internal coordinates. In these calculations, we
 used a Vanderbilt ultrasoft pseudopotential³³ with
 Perdew-Burke-Ernzerhof³⁴ (PBE) exchange-correlation
 functional and nonlinear core correction, which included
 semicore $3s3p$ states as well as $4s3d$ states in valence states.

IV. ENTHALPIES

We have calculated enthalpy $H(P)$ curves for each struc-
 ture in the pressure range 40–220 GPa based on density-
 functional methods^{35,36} using the PWSCF code.³⁰ Several en-
 ergy differences and relaxations were checked with the

TABLE I. Detailed structural data of the $\bar{I}43m$, $Pnma$, $Cmca$, and $P4_32_12$ Ca. (SG: space group; WP: Wyckoff position; and AC: atomic coordinates.)

SG	No.	WP	AC	x	y	z
$\bar{I}43m$	217	8c	(x, x, x)	~ 0.2		
$Pnma$	62	4c	$(x, 1/4, z)$	~ 0.3		~ 0.6
$Cmca$	64	8f	$(0, y, z)$		~ 0.3	~ 0.2
$P4_32_12$	96	8b	(x, y, z)	~ 0	~ 0.3	~ 0.3

182 QBOX,²⁹ FPLO,²⁷ and WIEN2K (Ref. 28) codes. In the 40–70
 183 GPa range, all five of the structures we have studied have
 184 enthalpies that differ by less than 20 meV/Ca (230 K/Ca), as
 185 shown in Fig. 2. In the 80–100 GPa range, the $P4_32_12$ phase
 186 is marginally the more stable phase. Three phases are degenerate,
 187 again within 20 meV/Ca, in the 100–130 GPa region
 188 and are almost exactly degenerate around 110–115 GPa.
 189 Thus at room temperature all five phases, including the sc
 190 one, are thermodynamically accessible up to 80–90 GPa,
 191 above which the sc and $\bar{I}43m$ structures become inaccessible.
 192 The other three phases remain thermally accessible to 130
 193 GPa. Above 140 GPa, the $Pnma$ phase becomes increasingly
 194 more stable than the others.

195 Our results agrees well with the results reported recently
 196 by Yao *et al.*²³ and Ishikawa *et al.*²⁴ in their corresponding
 197 pressure range. At low pressure, our result is apparently different
 198 from the result by Arapan *et al.*²⁵ In their results, sc Ca
 199 has the lowest enthalpy from 40 to 77 GPa, lower than the
 200 $P4_32_12$ and $Cmca$ structures. A possible reason is that the
 201 authors might not have taken into account the change in
 202 shape and internal coordinates of the $Cmca$ structure in the
 203 70–80 GPa pressure range. In our calculation, $b/a=1.0003$
 204 and internal coordinates $y=0.254$ and $z=0.225$ at 70 GPa
 205 (and similarly below) change dramatically to $b/a=1.0594$,
 206 $y=0.349$, and $z=0.199$ at 80 GPa (and similarly above).

207 Although equally dense, quasidegenerate, and related to
 208 the sc structure these structures differ in important ways
 209 from the sc structure and each other. In Fig. 1 the distribution
 210 of (first and second) neighbor distances d , relative to the sc
 211 lattice constant a_{sc} , are pictured. The collection of distances

cluster around $d/a_{sc} \sim 0.97-1.05$ and, more broadly, around
 $\sqrt{2}$. In an ensemble of nanocrystallites of these phases, the
 radial distribution function in the simplest picture should
 look like a broadened version of the sc one. For Ca the actual
 microscopic configuration at room temperature, where fluctuations
 (spacial and temporal) can occur among these phases (whose
 enthalpies differ by less than $k_B T_R$ per atom), will no doubt
 be much more complex. However, this simplistic radial distribution
 plot makes it plausible that the resulting thermal and spatial
 distribution of Ca atoms will produce an XRD pattern more like
 simple cubic than any other simple possibility. Teweldeberhan
 and Bonev¹⁵ have noted the near degeneracy of some of these
 phases in the 40–80 GPa region, and suggest that the $T=0$
 structure is $Pnma$ in the 45–90 GPa range, which is consistent
 with our results if the $P4_32_12$ structure is not included.

V. STABILITY AND LATTICE DYNAMICS

The structural stability of the (quasidegenerate) structures we
 have studied provide insight into behavior of Ca under pressure.
 Linear-response calculations were performed using the LMTART
 code^{31,32} to evaluate EPC.

60–100 GPa. The $\bar{I}43m$ and $Pnma$ structures are mostly
 dynamically stable from 60–100 GPa according to our linear-
 response calculations, but there are very soft zone boundary
 modes that verge on instability (small imaginary frequencies)
 at some pressures. The $Cmca$ and $P4_32_12$ structures are
 unstable over this entire pressure range; note that their
 structures are close to the sc structure. However, they are close
 to stable with very soft phonons at 100 GPa, where they were
 distorted far enough from the sc structure.

A rather common feature among these structures in this
 pressure range is softening of modes at the zone boundary,
 with associated low-frequency weight in the spectral function
 $\alpha^2(\omega)$ that can be seen in Figs. 3 and 4. Such low-
 frequency weight contributes strongly to λ , though the
 contribution to T_c is better judged⁷ by $\langle \omega \rangle \lambda$ or even
 $\langle \omega^2 \rangle \lambda$. With increase in pressure, the peaks move toward
 lower frequency, λ increases, and the structures approach
 instability. These results are consistent with the changes in
 structure parameters we obtain in the process of calculating the
 enthalpies, where all four structures evolve further from the
 sc structure with increase in pressure.

Above 100 GPa. At the highest pressures studied (by us,
 and experimentally), the crystal structures deviate more
 strongly from the sc structure. Of the structures we have
 considered, the $P4_32_12$ one becomes favored and also is

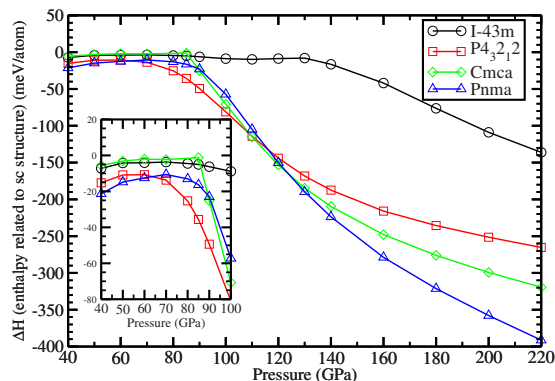


FIG. 2. (Color online) Plot of the enthalpy $H(P)$ of the four distorted Ca structures relative to that for Ca in the simple cubic structure. The inset gives an expanded picture of the 40–100 GPa regime.

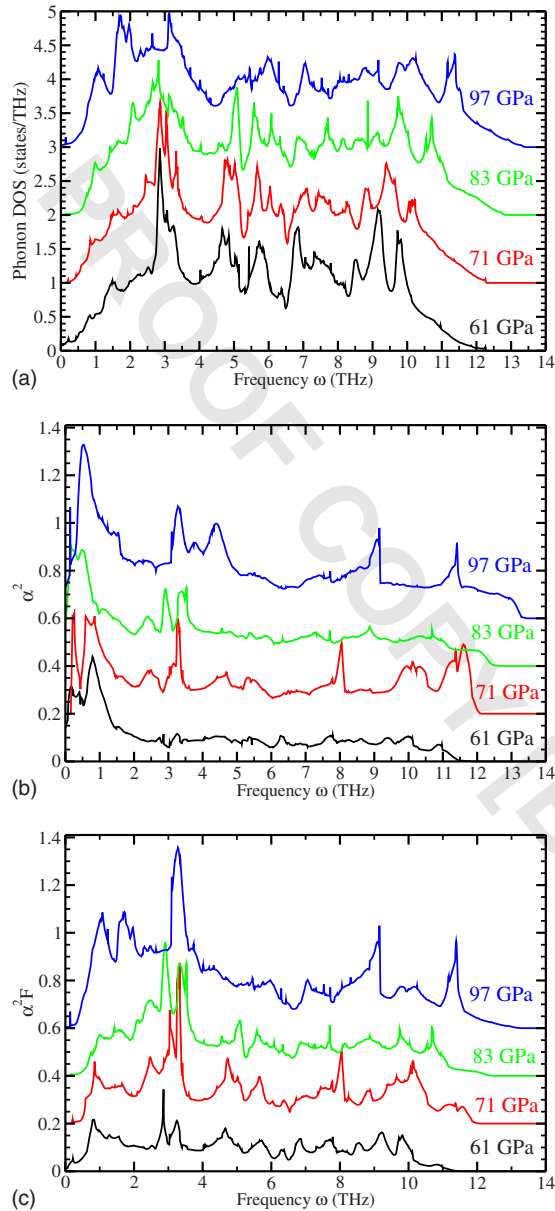


FIG. 3. (Color online) Plot of $\alpha^2 F(\omega)$ (lower panel), $\alpha^2(\omega)$ (middle panel), and phonon DOS (upper panel) of $I43m$ structure at about 61, 71, 83, and 97 GPa. This regime is characterized by strong coupling $\alpha^2(\omega)$ at very low frequency.

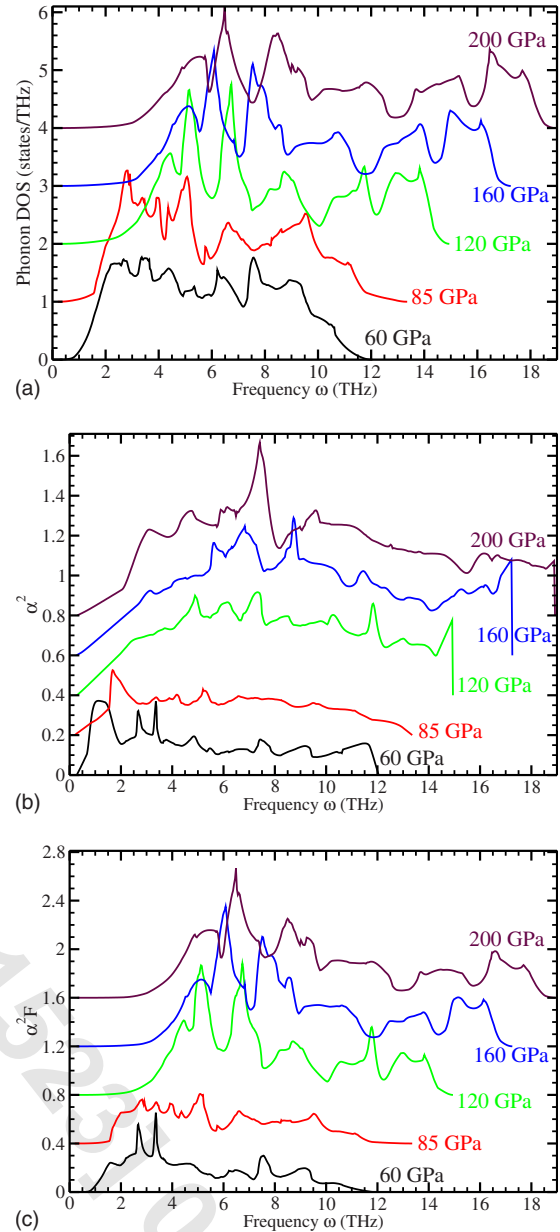


FIG. 4. (Color online) Plot of $\alpha^2 F(\omega)$ (bottom panel), $\alpha^2(\omega)$ (middle panel), and phonon DOS (upper panel) of $Pnma$ structure at about 60, 85, 120, 160, and 200 GPa. The main trends are the stiffening of the modes with increasing pressure, and the retention of coupling strength $\alpha^2(\omega)$ over a wide frequency range.

258 structurally stable around 110 GPa. This stability is consistent with the observed transition from the sc structure to the 259 Ca-IV structure at room temperature. The dramatic drop in 260 the electrical resistance at around 109 GPa is also consistent 261 with a transition from a locally disordered phase to a crystalline material.⁹ 262 263
 264 In the pressure range of 110–140 GPa, the $P4_32_12$, $Cmca$, 265 and $Pnma$ structures become quasidegenerate again. Linear- 266 response calculations of the $Pnma$ structure at 120 GPa and 267 above and of the $Cmca$ structure at around 130 GPa indeed 268 show strong coupling with $\lambda > 1.0$ in all the cases. Unlike 269 what was found below 100 GPa, there are no longer very 270 low-frequency phonons (see Figs. 4 and 5). The coupling

271 strength is spread over frequency, peaking for mid-range frequency phonons. 272
 273 Another interesting feature arises in the $\alpha^2(\omega)$ curves, 274 which reveal that the coupling matrix elements become relatively 275 uniform across most of the frequency range (except the uninteresting 276 acoustic modes below 2 THz) at pressures over 120 GPa in $Pnma$ structure and at 130 GPa in $Cmca$ 277 structure; this behavior is evident in Fig. 4 and especially in 278 Fig. 5 where the results for the $Cmca$ structure at 130 GPa 279 are pictured. This characteristic is fundamentally different 280 from that below 100 GPa, discussed above. 281
 282 At pressures over 140 GPa, the $Pnma$ structure is clearly 283 favored in our calculation, and linear-response calculations

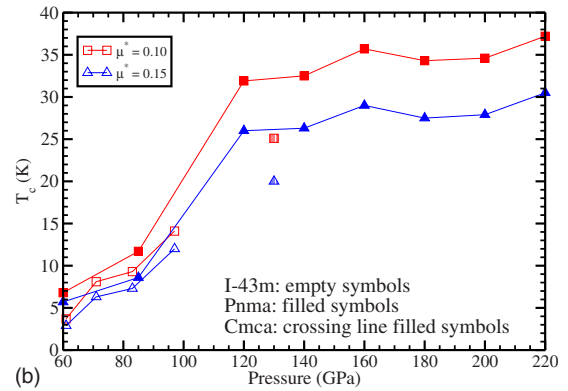
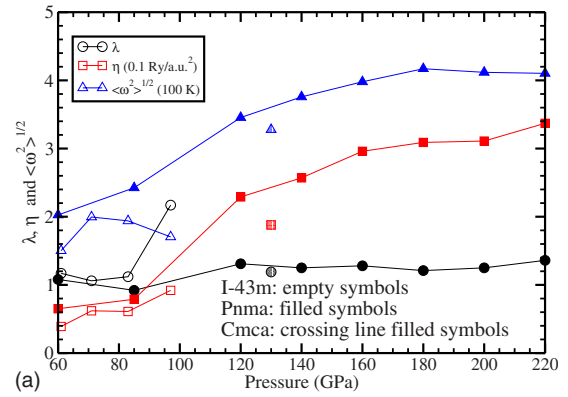
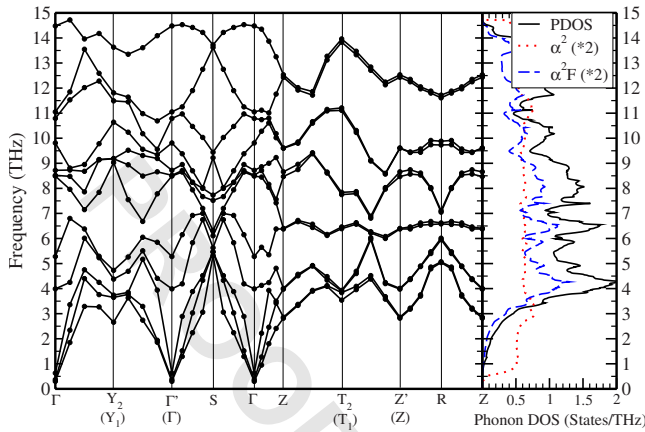


FIG. 6. (Color online) Upper panel: calculated electron-phonon coupling constant λ , η , and T_c of Ca in $I\bar{4}3m$ (empty symbols), *Pnma* (filled symbols), and *Cmca* (crossing-line filled symbols) structures at a few pressures. Lower panel: T_c calculated from the Allen-Dynes equation, showing the dependence on the Coulomb pseudopotential for which two values, $\mu^* = 0.10$ and 0.15 have been taken.

EPC constant λ is ~ 1.2 – 1.4 and the calculated T_c increases modestly from 25 – 30 K at 120 GPa to 30 – 35 K at 220 GPa. Neither the structure dependence nor the pressure dependence seems very important: the strong coupling and high T_c is more the rule than the exception. Ca at high pressure may be an excellent superconductor regardless of its structure.

VII. SUMMARY

Calculations of enthalpy versus pressure for five crystal-line phases of Ca (simple cubic and four distortions from it) indicate quasidegeneracy, with enthalpy differences small enough that one might expect a locally disordered, highly anharmonic, fluctuating structure at room temperature. Over most of the 30 – 150 GPa range, we find at least three crystal phases whose enthalpies indicate they will compete strongly at room temperature. The sc phase itself is badly unstable dynamically (at $T=0$), but the observed sc diffraction pattern can be understood as a locally noncrystalline, highly anharmonic phase derived from a spatially inhomogeneous and dynamically fluctuating combination of these structures, with most of them being straightforward distortions from the sc structure. Such a scenario seems to account qualitatively for the XRD observations of a sc structure.

284 indicate the structure is dynamically stable. The overall re-
285 sults are evident in Fig. 4, which shows that the structures
286 remain stable (no imaginary frequencies) and the lattice stiff-
287 ens smoothly with increasing pressure, and Fig. 6 shows that
288 strong electron-phonon coupling persists and T_c remains
289 high. In this high-pressure range, the incommensurate struc-
290 ture proposed by Arapan *et al.*²⁵ at pressure over 130 GPa is
291 also a possibility.

VI. COUPLING STRENGTH AND T_c

293 Figure 6 shows the calculated λ , $\eta = M_{Ca} \langle \omega^2 \rangle \lambda$, and rms
294 frequency $\langle \omega^2 \rangle^{1/2}$ versus pressure for a few structures and
295 pressures. The calculated values of T_c are shown in the lower
296 panel, using two values of Coulomb pseudopotential μ^*
297 $= 0.10$ and 0.15 that bracket the commonly used values and
298 therefore give an indication of the uncertainty due to the lack
299 of knowledge of the value of μ^* and its pressure dependence.
300 Results are provided for Ca in $I\bar{4}3m$, *Pnma*, and *Cmca* struc-
301 tures at a few pressures up to 220 GPa. In elemental metals
302 and in compounds where coupling is dominated by one atom
303 type, η has often been useful in characterizing contributions
304 to T_c .³⁷ η increases with pressure monotonically by a factor
305 of more than 5 from 60 to 220 GPa. The coupling constant λ
306 increases modestly up to 120 GPa then remains nearly con-
307 stant at $\lambda = 1.2$ – 1.4 . As pointed out elsewhere,³⁸ a dense zone
308 sampling is needed to calculate λ accurately, so any small
309 variation is probably not significant. The increase in η be-
310 yond 120 GPa correlates well with the lattice stiffening (in-
311 crease in $\langle \omega^2 \rangle$) in this pressure range.
312 The trend of the resulting T_c generally follows, but seems
313 to overestimate somewhat, the experimental values.⁹ For
314 *Cmca* structure at about 130 GPa, the calculated EPC
315 strength is $\lambda = 1.2$ and $T_c = 20$ – 25 K (for the two values of
316 μ^*) in very satisfactory agreement with the observed values
317 of T_c in this pressure range. For *Pnma* structure, T_c increases
318 rapidly in the 80 – 120 GPa region. At pressures above 120
319 GPa up to the maximum 220 GPa that we considered, the

342 At pressures below 100 GPa, the quasidegenerate struc-
 343 tures tend to have soft branches or occasionally lattice insta-
 344 bilities, which are associated with strong electron-phonon
 345 coupling. In the pressure range of 110–130 GPa three phases
 346 ($P4_32_12$, $Cmca$, and $Pnma$) again become quasidegenerate,
 347 and again it seems likely there will be spacial and temporal
 348 fluctuations between the structures. Of course other struc-
 349 tures may come into play as well; Arapan *et al.*²⁵ have pro-
 350 posed that the $Pnma$ structure competes with an incommen-
 351 surate structure at high pressure.

352 As our other main result, we find that linear-response cal-
 353 culation of the EPC strength and superconducting T_c ac-
 354 counts for its impressive superconductivity in the high-
 355 pressure regime and accounts in a broad sense for the strong
 356 increase in T_c in the sc phase. At higher pressure beyond the
 357 current experimental limit (i.e., 161 GPa), T_c still lies in the
 358 20–30 K range for some phases that we have studied. In fact
 359 strong electron-phonon coupling seems to be present in sev-
 360 eral phases across a substantial high-pressure range, although
 361 we have no simple picture why such strong coupling should
 362 arise. (The strong coupling in Li and Y likewise has no sim-
 363 ply physical explanation.^{6,7}) These results may resolve some
 364 of the perplexing questions on the structure and record high
 365 T_c for an element and should help in obtaining a more com-
 366 plete understanding of the rich phenomena that arise in
 367 simple metals at high pressure.

After submission of our manuscript, we became aware of 368
 a study by Yao *et al.*³⁹ They performed structural studies of 369
 calcium in the range 34–78 GPa using metadynamics and 370
 genetic algorithm methods. Their methods and results are 371
 complementary to ours, with each approach providing its 372
 own insights. Connections of their work to ours is evident, 373
 for example, the $I4_1/amd$ structure they focused on is 374
 slightly distorted from simple cubic, as are the structures that 375
 we study. Since its enthalpy is within 20 meV/Ca of the 376
 $Pnma$ structure across this pressure range, their result is con- 377
 sistent with our explanation of the observation of the simple 378
 cubic diffraction pattern at room temperature. Their linear- 379
 response calculations of electron-phonon coupling and the 380
 resulting T_c are also consistent with the more extensive re- 381
 sults that we present. 382

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